Pareto Improving Tax Reform

Michael Sampson
Department of Economics
Concordia University
1455 de Maisonneuve Blvd. W.
Montreal, Quebec
Canada H3G 1M8
Email: michael.sampson@concordia.ca

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Abstract

A computable general equilibrium model with taxation is constructed with heterogeneity in tastes, technology, and endowments. The equilibrium is shown to be unique and easy to compute. Pareto efficiency requires a uniform rate of taxation. Given an inefficient tax regime, it is shown how to implement an efficient uniform taxation regime where everybody in the economy is made strictly better off. A numerical example is included.
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1 Introduction

A traveler seeking directions does not want to know what the destination looks like, but how to get there. A similar problem exists with the optimal taxation literature. Beginning with Diamond and Mirrlees (1971), it does a good job of characterizing optimal taxation, but a poor job of explaining how to get there. Atkinson and Stiglitz (1976) show that a uniform taxation is optimal, but they do not show how to implement uniform taxation without running in to a hornets' nest of political problems. Heterogeneity is the key. Simply implementing uniform taxation will generally hurt those who benefit from low tax rates on the goods they consume, or who obtain above average government benefits. Heterogeneity extends to governments as well: often the most difficult political problems are with different levels of government disagreeing on how tax revenue should be divided or how different goods should be taxed. When these various interests come into the political arena they will have an incentive to block the implementation of any tax reform that hurts their interests, even if the new tax regime can be shown to be efficient.

In this paper we will try to give our weary traveler directions. We show how implement optimal tax reform in a way that makes everyone in the economy better off, including the various levels of government, and so avoid these political difficulties. We construct a computable general equilibrium model with taxation which, within the assumed functional forms, allows for unrestricted heterogeneity in tastes, technology, and endowments. The model is general enough to allow an unlimited number of levels of government, and so the optimal tax regime can be derived in a way that respects the competing interests of the various levels of government. Computation is easy, requiring nothing more than matrix multiplication or matrix inversion.

Section 2 presents the basic model without taxation in order to establish notation and baseline results. In Section 3 we add taxation to the model and show that the resulting equilibrium is inefficient if taxation is not uniform. In Section 4 we show how to design and compute a uniform taxation structure that makes everybody in the economy strictly better off. In Section 5 an illustrative numerical example is presented.

2 The Model

The economy has a set of households $\mathcal{H} = \{1, \ldots, H\}$ indexed by i, a set of goods $\mathcal{G} = \{1, \ldots, G\}$ indexed by j, and a set of factors of production $\mathcal{F} = \{1, \ldots, F\}$ indexed by l. The quantity and price of good $j \in \mathcal{G}$ are Q_j and P_j. The total supply of factor $l \in \mathcal{F}$ is E_l with factor price W_l. We adopt the normalization $E_l \equiv 1$ for all $l \in \mathcal{F}$.

Total expenditure is $Y \equiv \sum_{j=1}^{G} P_j Q_j$, the expenditure share of good j is $v_j \equiv \frac{P_j Q_j}{Y}$, and we define the $1 \times G$ vector v as $v \equiv [v_j]$. Define $\omega_l \equiv \frac{W_l E_l}{Y}$, the $1 \times F$ vector $\omega \equiv [\omega_l]$, and let $\iota_n \equiv [1]$ denote an $n \times 1$ vector of ones. We normalize prices so that $Y \equiv 1$ so that

$$v \iota_G = 1. \tag{1}$$

Household $i \in \mathcal{H}$ has an endowment $E_{li} > 0$ of factor $l \in \mathcal{F}$. Since

$$\sum_{i=1}^{H} E_{li} = E_l = 1$$

the $H \times F$ matrix $E \equiv [E_{li}]$ has rows that sum to one and so

$$E\iota_H = \iota_F. \tag{2}$$

Income of household i is $Y_i \equiv \sum_{l=1}^{F} \omega_l E_{li}$ with $y_i \equiv \frac{Y_i}{Y}$ being household $i's$ share of total income Y. Since $Y \equiv 1$ we have $Y_i = y_i$. Define the $1 \times H$ vector $y \equiv [y_i]$ so that

$$y = \omega E. \tag{3}$$

Household i consumes Q_{ij} of good $j \in \mathcal{G}$ with utility function[1]

$$U_i = \sum_{j=1}^{G} \alpha_{ij} \ln(Q_{ij}) \text{ with } \alpha_{ij} > 0 \text{ and } \sum_{j=1}^{G} \alpha_{ij} = 1. \tag{4}$$

The $H \times G$ matrix $A \equiv [\alpha_{ij}]$ has rows that sum to one as

$$A\iota_G = \iota_H. \tag{5}$$

Utility maximization requires that $P_j Q_{ij} = y_i \alpha_{ij}$ so that if we define household $i's$ share of good j as $q_{ij} \equiv \frac{Q_{ij}}{Q_j}$ then

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}. \tag{6}$$

Summing over i and using $\sum_{i=1}^{H} q_{ij} = 1$ yields $v_j = \sum_{i=1}^{H} y_i \alpha_{ij}$ or

$$v = yA. \tag{7}$$

Good $j \in \mathcal{G}$ is produced by a constant returns to scale Cobb-Douglas production function

$$Q_j = \prod_{l=1}^{F} K_{jl}^{\beta_{jl}} \text{ with } \beta_{jl} > 0 \text{ and } \sum_{l=1}^{F} \beta_{jl} = 1.$$

[1] Pure leisure is a special case where some factor l and good j are interpreted as time, where $\beta_{jl} = 1$, and so $Q_j = K_{jl}$.

The $G \times F$ matrix $B \equiv \left[\beta_{jl}\right]$ then has rows that sum to 1 or

$$B\iota_F = \iota_G. \tag{8}$$

The first-order conditions for profit maximization require that

$$\frac{W_l K_{jl}}{P_j Q_j} = \beta_{jl} \text{ or } K_{jl} = \frac{v_j \beta_{jl}}{\omega_l}.$$

Summing over j and using $\sum_{j=1}^{G} K_{jl} = 1 = E_l$ yields $\omega_l = \sum_{j=1}^{G} v_j \beta_{jl}$ or

$$\omega = vB. \tag{9}$$

The normalization of prices as $v\iota_G = 1$ also applies to the vectors ω, y since

$$\begin{aligned} \omega\iota_F &= vB\iota_F = v\iota_G = 1 \\ y\iota_H &= \omega E\iota_H = \omega\iota_F = 1. \end{aligned}$$

To find the competitive equilibrium we solve for the v, ω, y that satisfy

$$\omega = vB, v = yA, y = \omega E \text{ with } v\iota_G = \omega\iota_F = y\iota_H = 1. \tag{10}$$

Combining the three relations in (10) we have

$$y = yC \text{ where } C \equiv ABE. \tag{11}$$

Since A, B, E have strictly positive elements, it follows that the elements of the $H \times H$ matrix C are also strictly positive. Furthermore C has rows that sum to one since

$$C\iota_H = ABE\iota_H = AB\iota_F = A\iota_G = \iota_H.$$

By the Perron-Frobenius theorem (see Grimmett, and Stirzaker (1982)) there exists a strictly positive and unique vector y satisfying $y\iota_H = 1$ that solves $y = yC$. Given y we can then solve for $v = yA$ and $\omega = vB$, which are also strictly positive. This leads to the competitive allocation that we denote as $\{Q, K\}$ where the $H \times G$ matrix $Q \equiv [q_{ij}]$ and the $G \times F$ matrix $K \equiv [K_{jl}]$ are given by

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}, K_{jl} = \frac{v_j \beta_{jl}}{\omega_l}. \tag{12}$$

In the allocation $\{Q, K\}$ the matrix Q determines how a given level of production is allocated amongst households, while the matrix K determines how factor endowments are allocated amongst industries.

The assumption of strictly positive elements of A, B, E can be weakened considerably at the cost of complicating some of the proofs. As shown in Sampson (2013), it is possible to insure strictly positive prices and incomes with zero elements A, B, E as long as 1) the economy is interconnected enough so that the matrix C is indecomposable, 2) all goods are desired by a least one person and 3) all factors are needed in the production of some good.

2.1 Computation

We discuss two methods of computing the fixed point of $y = yC$ for $y\iota_H = 1$. Neither method requires anything more difficult than matrix multiplication or matrix inversion. The first method is iterative, does not require matrix inversion, and is probably best for large models. Let the $1\times H$ vector y^t be the t^{th} iteration where

$$y^t = y^{t-1}C$$

with any starting value y^0 satisfying $y^0\iota_H = 1$. Since C is a positive matrix with rows that sum to 1, C has a one eigenvalue $\lambda_1 = 1$ with $|\lambda_i| < 1$ for $i = 2, \ldots H$. It follows that $y^t \to y$ exponentially at a rate determined by the second largest eigenvalue in absolute value.

The second method is to directly solve the linear equations $y = yC$ and $y\iota_H = 1$ for y. To this end partition C and y as

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \text{ and } y = \begin{bmatrix} y_1 & y_2 \end{bmatrix}$$

where C_{22} and y_2 are scalars. Since $C_{12} \neq 0$ it follows that $(I - C_{11})^{-1} = \sum_{k=0}^{\infty} C_{11}^k$ exists and has strictly positive elements, and so

$$y_1 = \frac{C_{21}(I - C_{11})^{-1}}{1 + C_{21}(I - C_{11})^{-1}\iota_{H-1}}, y_2 = \frac{1}{1 + C_{21}(I - C_{11})^{-1}\iota_{H-1}}.$$

Once y is calculated we have $v = pA$ and $\omega = vB$, from which the allocation $\{Q, K\}$ in (12) can be calculated.

If the number of goods G or factors F is smaller than the number of households H, then the size of the problem can be accordingly reduced. Thus if the number of goods G is smaller than the number of households H, then by multiplying both sides of $y = yC$ by A and using $v = yA$ we can instead solve $v = vC_G$ where the $H \times H$ matrix $C \equiv ABE$ is replaced by the smaller $G \times G$ matrix $C_G \equiv BEA$, and where $C_G \iota_G = \iota_G$. If the number of factors F is smaller than the number of goods G, then by multiplying both sides of $v = vC_G$ by B and using $\omega = vB$ we can instead solve $\omega = \omega C_F$ where the $G \times G$ matrix C_G is replaced by the $F \times F$ matrix $C_F \equiv EAB$, and where $C_F \iota_F = \iota_F$.

3 Introducing Taxation

Consider a sales tax on firms of rate $0 \le \tau_j < 1$ so that firms selling good j and collecting revenue v_j must pay a tax of $v_j \tau_j$. (We assume $\tau_j > 0$ for at least one good.) Total tax revenue $\tau = \sum_{j=1}^{G} v_j \tau_j$ is then redistributed to households with household i receiving an exogenous share $\bar{y}_i \ge 0$ with $\sum_{i=1}^{H} \bar{y}_i = 1$ so that household income is

$$y_i = \sum_{l=1}^{F} \omega_l E_{li} + \tau \bar{y}_i \text{ or } y = \omega E + \tau \bar{y}.$$

Here we note that, suitably interpreted, the model is general enough to include one or more levels of selfish governments, each with its own utility function. To do this we merely identify a subset $\mathcal{H}_G \subseteq \mathcal{H}$ of households as governments. For example $i_1 \in \mathcal{H}_G$ might be a particular state government that raises its revenue y_{i_1} from its tax share $\tau \bar{y}_{i_1}$ and endowments E_{li_1} for $l \in \mathcal{F}$ (for example oil in the ground). It then spends its revenue y_{i_1} on goods and services $Q_{i_1 j}$ for $j \in \mathcal{G}$ in a manner that maximizes its utility U_{i_1}. Yet another $i_2 \in \mathcal{H}_G$ might be the federal government, and another $i_3 \in \mathcal{H}_G$ a municipal government.

Profits π_j are

$$\pi_j = P_j Q_j (1 - \tau_j) - \sum_{l=1}^{F} \omega_l K_{jl} = v_j (1 - \tau_j) - \sum_{l=1}^{F} \omega_l K_{jl}.$$

The first-order conditions for profit maximization then imply that

$$\frac{\omega_l K_{jl}}{P_j Q_j (1 - \tau_j)} = \beta_{jl} \text{ or } K_{jl} = \frac{v_j (1 - \tau_j) \beta_{jl}}{\omega_l}.$$

Summing over j and using $\sum_{j=1}^{G} K_{jl} = E_l = 1$ yields

$$\omega = v\left(I - \tau_d\right)B$$

where $\tau_d \equiv diag\left[\tau_1, \tau_2, \ldots, \tau_G\right]$ is a $G \times G$ diagonal matrix.

With the new tax regime added the model now becomes

$$y = \omega E + \tau\bar{y},\ \ v = yA, \omega = v\left(I - \tau_d\right)B \tag{13}$$

with $\bar{y}\iota_H = 1$ and $\tau = v\tau_d\iota_G$. The normalization $v\iota_G = 1$ then implies $y\iota_H = 1$ and $\omega\iota_F = 1 - \tau$ as

$$1 = v\iota_G = yA\iota_G = y\iota_H \text{ and } \omega\iota_F = v\left(I - \tau_d\right)B\iota_F = v\left(I - \tau_d\right)\iota_G = 1 - \tau.$$

From (13) we have $y = yC_{\tau_d}$ as

$$y = yA\left(I - \tau_d\right)BE + yA\tau_d\iota_G\bar{y} = y\left(A\left(I - \tau_d\right)BE + \tau_d\iota_G\bar{y}\right) = yC_{\tau_d}$$

where

$$C_{\tau_d} = A\left(I - \tau_d\right)BE + A\tau_d\iota_G\bar{y}.$$

We have $C_{\tau_d}\iota_H = \iota_H$ as

$$\begin{aligned} C_{\tau_d}\iota_H &= \left(A\left(I - \tau_d\right)BE + A\tau_d\iota_G\bar{y}\right)\iota_H = ABE\iota_H - A\tau_d BE\iota_H + A\tau_d\iota_G \\ &= \iota_H - A\tau_d\iota_G + A\tau_d\iota_G = \iota_H. \end{aligned}$$

Thus the problem of solving for y in $y = yC_{\tau_d}$ for $y\iota_H = 1$ is equivalent to solving for $y = yC$ in Section 2.1, and so y can be found using either of the two methods discussed in Section 2.1.

Assuming that $\tau_d \neq 0$ we can obtain an expression for y without having to partition C_{τ_d}. From (13) we have

$$y = \omega E + \tau\bar{y} = v\left(I - \tau_d\right)BE + \tau\bar{y} = yA\left(I - \tau_d\right)BE + \tau\bar{y} \tag{14}$$

so

$$y = \tau\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}.$$

This expression is valid since $0 \leq \tau_j < 1$ with $\tau_j > 0$ for at least one j, and so the matrix $A\left(I - \tau_d\right)BE$ has eigenvalues strictly less than one in modulus and hence the matrix

$$\left(I - A\left(I - \tau_d\right)BE\right)^{-1} = I + \sum_{m=1}^{\infty}\left(A\left(I - \tau_d\right)BE\right)^m$$

exists and is strictly positive. Since $y\iota_H = 1$ we can solve for tax revenue τ and y as

$$\tau = \left(\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}\iota_H\right)^{-1} \text{ and } y = \frac{\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}}{\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}\iota_H}. \tag{15}$$

Once we have y we can then obtain $v = yA$ and $\omega = v\left(I - \tau_d\right)B$, leading to an allocation $\{Q, K\}$ where

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}, K_{jl} = \frac{v_j\left(1 - \tau_j\right)\beta_{jl}}{\omega_l}. \tag{16}$$

3.1 Uniform Taxation

Uniform taxation occurs when $\tau_d = \tau I$ where the scalar τ is the uniform tax rate. Uniform taxation leads to a Pareto efficient allocation since with $\tau_d = \tau I$ in (14) we obtain

$$y = yABE\left(1 - \tau\right) + \tau\bar{y} = yAB\left(E\left(1 - \tau\right) + \tau\iota_F\bar{y}\right) = yABE_\tau = yC_\tau$$

where

$$E_\tau = E\left(1 - \tau\right) + \tau\iota_F\bar{y} \text{ and } C_\tau = ABE_\tau.$$

That is, uniform taxation is equivalent changing the endowment matrix from E to E_τ.

4 Tax Reform

The problem we address in this section is how to reform an inefficient tax system by implementing a uniform tax τ^* and vector of tax shares $\bar{y}^*$ in such a way that everybody in the economy benefits.

4.1 Efficiency

Our first task is to characterize any Pareto optimal allocation $\{Q^*, K^*\}$. We exclude allocations where some households have zero consumption so that we require $U_i^* > -\infty$ for all $i \in \mathcal{H}$. Given a $1 \times H$ vector $y^* \equiv [y_i^*]$ satisfying $y_i^* > 0$ (this insures $U_i^* > -\infty$) and $y^* \iota_H = 1$, consider maximizing the social welfare function

$$U^* \equiv \sum_{i=1}^{H} y_i^* U_i = \sum_{i=1}^{H} \sum_{j=1}^{G} y_i^* \alpha_{ij} \ln(q_{ij}) + \sum_{j=1}^{G} \sum_{l=1}^{F} v_j^* \beta_{jl} \ln(K_{jl}) \tag{17}$$

subject to the feasibility constraints

$$\sum_{i=1}^{H} q_{ij} = 1 \text{ for all } j \in \mathcal{G} \text{ and } \sum_{j=1}^{G} K_{jl} = 1 \text{ for all } l \in \mathcal{F} \tag{18}$$

where $v^* \equiv \left[v_j^*\right] = y^* A$ and $\omega^* \equiv [\omega_l^*] = v^* B$. We can rewrite (17) as

$$U^* \equiv \sum_{j=1}^{G} v_j^* \sum_{i=1}^{H} \frac{y_i^* \alpha_{ij}}{v_j^*} \ln(q_{ij}) + \sum_{l=1}^{F} \omega_l^* \sum_{j=1}^{G} \frac{v_j^* \beta_{jl}}{\omega_l^*} \ln(K_{jl})$$

so that using either calculus or an information inequality (see Rao, 1973, p 58), it follows that this has a unique solution, the allocation $\{Q^*, K^*\}$ given by

$$q_{ij}^* = \frac{y_i^* \alpha_{ij}}{v_j^*}, K_{jl}^* = \frac{v_j^* \beta_{jl}}{\omega_l^*}. \tag{19}$$

Since $\{Q^*, K^*\}$ maximizes U^*, it follows that any allocation of the form (19) is Pareto optimal. It follows then that the competitive allocation in (12) with

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}, K_{jl} = \frac{v_j \beta_{jl}}{\omega_l}$$

is Pareto optimal with $y^* = y, v^* = v$, and $\omega^* = \omega$. As well the uniform taxation allocation in (16) given by

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}, K_{jl} = \frac{v_j \beta_{jl} (1 - \tau)}{\omega_l}$$

is Pareto optimal with $y^* = y, v^* = v$, and $\omega^* = \frac{\omega}{1-\tau}$.

4.2 Making Everybody Better Off

Our next task is to show how to calculate a Pareto efficient allocation that makes everybody in the economy better off relative to an inefficient status quo allocation. Denote the inefficient status quo allocation $\{Q, K\}$ satisfying

$$\sum_{i=1}^{H} q_{ij} = 1 \text{ and } \sum_{j=1}^{G} K_{jl} = 1$$

with household utility

$$U_i = \sum_{j=1}^{G} \alpha_{ij} \ln(q_{ij}) + \sum_{j=1}^{G} \sum_{l=1}^{F} \alpha_{ij}\beta_{jl} \ln(K_{jl}) \text{ for all } i \in \mathcal{H}.$$

Consider then a competitive allocation where we endow each household as

$$E^*_{li} \equiv \sum_{j=1}^{G} q_{ij} K_{jl} \text{ or } E^* \equiv [E^*_{li}] = K^T Q^T. \tag{20}$$

Replacing E with E^* in (11), we compute the corresponding competitive equilibrium as

$$y^* = y^* C^* \text{ where } C^* \equiv ABE^*, v^* = y^* A, \omega^* = v^* B. \tag{21}$$

This leads to the Pareto optimal competitive allocation $\{Q^*, K^*\}$ given by

$$q^*_{ij} = \frac{y^*_i \alpha_{ij}}{v^*_j}, K^*_{jl} = \frac{v^*_j \beta_{jl}}{\omega^*_l} \tag{22}$$

with utility

$$U^*_i = \sum_{j=1}^{G} \alpha_{ij} \ln\left(q^*_{ij}\right) + \sum_{j=1}^{G} \sum_{l=1}^{F} \alpha_{ij}\beta_{jl} \ln\left(K^*_{jl}\right) \text{ for all } i \in \mathcal{H}.$$

Here we show that moving to the allocation $\{Q^*, K^*\}$ makes everyone at least as well off as the status quo allocation $\{Q, K\}$.

Theorem 1 $U^*_i \geq U_i$ *for all* $i \in \mathcal{H}$.

Proof. With $Y^* \equiv 1$ we have $W_l^* = \omega_l^*, Y_i^* = y_i^*$ from (21) and

$$P_j^* = \frac{v_j^*}{Q_j^*} \text{ where } Q_j^* \equiv \prod_{l=1}^{F} \left(K_{jl}^*\right)^{\beta_{jl}}.$$

The production plan with inputs K_{jl} and output $Q_j \equiv \prod_{l=1}^{F} (K_{jl})^{\beta_{jl}}$ is feasible and hence must make non-positive profits, and so

$$\tilde{\pi}_j \equiv P_j^* Q_j - \sum_{l=1}^{F} \omega_l^* K_{jl} \leq 0 \tag{23}$$

with $\tilde{\pi}_j = 0$ if and only if $K_{jl} = K_{jl}^*$ and $Q_j = Q_j^*$. Define

$$\tilde{Y}_i \equiv \sum_{j=1}^{G} P_j^* Q_{ij}.$$

Multiplying both sides of (23) by q_{ij}, summing over j, and using $\tilde{\pi}_j \leq 0$ implies that $\tilde{Y}_i \leq y_i^*$ as

$$\tilde{Y}_i = \sum_{j=1}^{G} P_j^* q_{ij} Q_j = \sum_{j=1}^{G} q_{ij} \tilde{\pi}_j + \sum_{l=1}^{F} \omega_l^* \sum_{j=1}^{G} q_{ij} K_{jl} = \sum_{j=1}^{G} q_{ij} \tilde{\pi}_j + y_i^* \leq y_i^* \tag{24}$$

with equality if an only if $\tilde{\pi}_j = 0$ for all $j \in G$. Since $y_i^* \geq \tilde{Y}_i$, household i in the constructed competitive equilibrium can afford its status quo consumption Q_{ij} for all $j \in \mathcal{G}$, and so the desired result $U_i^* \geq U_i$ for all $i \in \mathcal{H}$ follows. ■

Here we show that having an allocation of the form $\{Q^*, K^*\}$ in (22) is necessary and sufficient for an allocation to be Pareto optimal.

Theorem 2 *An allocation $\{Q, K\}$ is Pareto optimal if and only if there exists a strictly positive $1 \times H$ vector y^* satisfying $y^* \iota_H = 1$ such that*

$$q_{ij} = \frac{y_i^* \alpha_{ij}}{v_j^*}, K_{jl} = \frac{v_j^* \beta_{jl}}{\omega_l^*} \text{ where } v^* = y^* A, \omega^* = v^* B.$$

Proof. We have already shown that an allocation of the above form is Pareto optimal. Now suppose that $\{Q, K\}$ is Pareto optimal. If we calculate y^* from (21) then it must be that $U_i = U_i^*$ for all $i \in \mathcal{H}$ since $\{Q, K\}$ is Pareto optimal. It follows that $\{Q, K\}$ maximizes the social welfare function U^* in (17) with income distribution y^*. Since this has a unique maximum the result follows. ■

We now prove that any non-uniform tax regime is inefficient.

Theorem 3 *The tax allocation* $\{Q, K\}$ *in (16) given by*

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j}, K_{jl} = \frac{v_j (1 - \tau_j) \beta_{jl}}{\omega_l}$$

is Pareto optimal if and only if $\tau_j = \tau$ *for all* $j \in \mathcal{F}$.

Proof. We have already shown that a uniform tax leads to a Pareto optimal allocation. Now suppose the allocation is Pareto efficient. From Theorem 2 there exists a y^* with $v^* = y^* A$ and $\omega^* = v^* B$ such that $q_{ij} = q_{ij}^*$ and $K_{jl} = K_{jl}^*$ or

$$\frac{y_i \alpha_{ij}}{v_j} = \frac{y_i^* \alpha_{ij}}{v_j^*} \text{ and } \frac{v_j (1 - \tau_j) \beta_{jl}}{\omega_l} = \frac{v_j^* \beta_{jl}}{\omega_l^*}.$$

Cancelling $\alpha_{ij} > 0$ from $q_{ij} = q_{ij}^*$ we have $y_i^* v_j = v_j^* y_i$. Summing over i and using

$$\sum_{i=1}^{H} y_i = \sum_{i=1}^{H} y_i^* = 1$$

yields $v_j^* = v_j$ for all $j \in \mathcal{G}$ and hence $y_i^* = y_i$ for all $i \in \mathcal{H}$. Cancelling $\beta_{jl} > 0$ from $K_{jl} = K_{jl}^*$ and using $v_j^* = v_j$ we have $(1 - \tau_j)\, \omega_l^* = \omega_l$. Summing over l and using

$$\sum_{l=1}^{F} \omega_l^* = 1 \text{ and } \sum_{l=1}^{F} \omega_l = 1 - \tau$$

yields $\tau_j = \tau$ for all $j \in \mathcal{G}$. ■

Suppose then that we have an inefficient tax regime $\{Q, K\}$ with non-uniform tax rates $\tau_d \neq \tau I$. We seek to replace it by an efficient uniform tax regime $\{Q^*, K^*\}$ with uniform tax rate τ^* and a corresponding $1 \times H$ vector of tax shares $\bar{y}^* = [\bar{y}_i^*]$ where $U_i^* > U_i$ for all $i \in \mathcal{H}$. From the inefficient initial allocation $\{Q, K\}$ define $E^* = [E_{li}^*]$ using (20) as

$$E_{li}^* \equiv \sum_{j=1}^{G} q_{ij} K_{jl} = \frac{y_i}{\omega_l} \sum_{j=1}^{G} \alpha_{ij} (1 - \tau_j) \beta_{jl}$$

with $C^* = ABE^*$. The resulting competitive equilibrium yields an allocation $\{Q^*, K^*\}$ given by

$$q_{ij}^* = \frac{y_i^* \alpha_{ij}}{v_j^*}, K_{jl}^* = \frac{v_j^* \beta_{jl}}{\omega_l^*} \text{ for all } i \in \mathcal{H}, j \in \mathcal{G}, l \in \mathcal{F}.$$

By construction this allocation satisfies $U_i^* \geq U_i$ for all $i \in \mathcal{H}$. Since the tax is not uniform and $\{Q, K\}$ is not Pareto efficient, it follows by Theorem 3 that $U_i^* > U_i$ for some $i \in \mathcal{H}$.

In fact we can show that $U_i^* > U_i$ for all $i \in \mathcal{H}$, that is the allocation $\{Q^* K^*\}$ will make everybody in the economy better off.

Theorem 4 *If the tax is not uniform then $U_i^* > U_i$ for all $i \in \mathcal{H}$.*

Proof. We demonstrate this by showing that if $U_i^* = U_i$ for one household, then the tax must be uniform. From (24) we have

$$\tilde{Y}_i \equiv \sum_{j=1}^{G} P_j^* Q_{ij} = \sum_{j=1}^{G} q_{ij} \tilde{\pi}_j + y_i^* \leq y_i^*.$$

If $U_i^* = U_i$ it must be that $\tilde{Y}_i = y_i^*$ since if $\tilde{Y}_i < y_i^*$ this household could consume its status quo consumption Q_{ij}, obtain utility U_i^*, and spend the remaining income $y_i^* - \tilde{Y}_i > 0$ to achieve a higher level of utility than U_i^*, which is impossible. Since Q_{ij}^* for $j \in \mathcal{G}$ is the unique consumption bundle that maximizes utility in the competitive equilibrium, it follows that $Q_{ij} = Q_{ij}^*$ for all $j \in \mathcal{G}$. Since $\tilde{Y}_i = y_i^*$ it follows that $\tilde{\pi}_j = 0, Q_j = Q_j^*$ and

$$K_{jl} = \frac{v_j (1 - \tau_j) \beta_{jl}}{\omega_l} = K_{jl}^* = \frac{v_j^* \beta_{jl}}{\omega_l^*} \text{ for all } j \in \mathcal{G}, l \in \mathcal{F}.$$

Thus $v_j (1 - \tau_j) \omega_l^* = v_j^* \omega_l$ and so summing over l and using

$$\sum_{l=1}^{F} \omega_l^* = 1, \ \sum_{l=1}^{F} \omega_l = 1 - \tau$$

yields

$$v_j = \frac{v_j^* (1 - \tau)}{(1 - \tau_j)} \text{ for all } j \in \mathcal{G}.$$

For the household with $U_i = U_i^*$ we have already shown that $Q_{ij} = Q_{ij}^*$. Since $Q_j = Q_j^*$ it follows that

$$q_{ij} = \frac{y_i \alpha_{ij}}{v_j} = \frac{y_i^* \alpha_{ij} (1 - \tau_j)}{v_j^* (1 - \tau)} = q_{ij}^* = \frac{y_i^* \alpha_{ij}}{v_j^*} \text{ for all } j \in \mathcal{G}$$

so that $(1-\tau_j)\, y_i = (1-\tau)\, y_i^*$. Summing over i and using

$$\sum_{l=1}^{F} y_i^* = \sum_{l=1}^{F} y_i = 1$$

then yields the required result $\tau_j = \tau$ for all $j \in \mathcal{G}$. ■

4.3 Implementing The Optimal Tax Regime

Optimal tax reform leading to the Pareto improving allocation $\{Q^*, K^*\}$ depends then on engineering tax policy so that we achieve the target optimal income distribution y^*. This can be done by choosing $\tau^*, \overline{y}^*$ according to Theorem 5 below. To this end note from (14) that with an optimal income distribution y^* and a uniform tax we have

$$y^* = y^* ABE\,(1-\tau^*) + \tau^* \overline{y}^* = \hat{y}\,(1-\tau^*) + \tau^* \overline{y}^* \tag{25}$$

where $\hat{y} = y^* ABE$ or

$$\hat{y}_i = \sum_{l=1}^{F} \omega_l^* E_{il} > 0 \text{ with } \omega^* \equiv y^* AB.$$

Here $\hat{y}_i$ is the income of household i if ω^* were used to value its endowments. Since $ABE\iota_H = \iota_H$ it follows that $\hat{y}\iota_H = 1$.

Theorem 5 *A given income distribution y^* can be achieved with a uniform tax τ^* satisfying $\tau^*_{\min} \leq \tau^* \leq 1$ where*

$$\overline{y}^* = \begin{cases} y^* & \text{if } \hat{y} = y^* \\ \frac{y^* - \hat{y}}{\tau^*} + \hat{y} & \text{if } \hat{y} \neq y^* \end{cases} \quad \text{and } \tau^*_{\min} = \begin{cases} 0 & \text{if } \hat{y} = y^* \\ 1 - \min_{i \in \mathcal{H}} \frac{y_i^*}{\hat{y}_i} & \text{if } \hat{y} \neq y^* \end{cases}.$$

Proof. First suppose that $\hat{y} = y^*$. From (25) it follows that the government can achieve the desired distribution of income y^* by abolishing all taxes and subsidies and setting $\tau^* = 0$. More generally it can chose any tax rate $0 \leq \tau^* \leq 1$ in (25) as long as it sets $\overline{y}^* = y$. Now suppose that $\hat{y} \neq y^*$. For a given $\tau^* > 0$ we can achieve y^* by setting $\overline{y}^* = \frac{1}{\tau^*}(y^* - \hat{y}) + \hat{y}$ or

$$\overline{y}_i^* = \frac{y_i^* - \hat{y}_i}{\tau^*} + \hat{y}_i. \tag{26}$$

If $y_i^* > \hat{y}_i$ then any tax rate $0 < \tau^* \leq 1$ will insure that $\bar{y}_i \geq 0$. If $y_i < \hat{y}_i$, as it must be for at least one household since $\hat{y} \neq y^*$ and $(\hat{y} - y^*)\,\iota_H = 0$, then to insure that $\bar{y}_i \geq 0$ we require

$$\tau^* \geq \frac{\hat{y}_i - y_i^*}{\hat{y}_i} = 1 - \frac{y_i^*}{\hat{y}_i}.$$

The minimum tax rate $0 < \tau_{\min} < 1$ that insures $\bar{y}_i \geq 0$ for all $i \in \mathcal{H}$ is then

$$\tau_{\min} \equiv \max_{i \in \mathcal{H}} \frac{\hat{y}_i - y_i^*}{\hat{y}_i} = 1 - \min_{i \in \mathcal{H}} \frac{y_i^*}{\hat{y}_i}. \tag{27}$$

■

Notice that unless $\hat{y} = y^*$ the exercise of imposing a Pareto improving tax reform puts a lower bound on the size of government, as given by $\tau_{\min}$.

4.4 Perturbing y^*

The allocation $\{Q^*, K^*\}$ with income distribution y^* leading to $U_i^* > U_i$ for all $i \in \mathcal{H}$ is not unique. Even though all households benefit, some households might benefit more than others, and so one might want to consider alternative allocations. These can be explored by considering a first-order perturbation around y^* of $y^* + \delta\Delta y^*$, where $\delta \geq 0$ is a sufficiently small scalar and Δy^* is any $1 \times H$ vector of directions satisfying $\Delta y^* \iota_H = 0$.

Let the utility of household i with this perturbed income distribution be $U_i^* + \delta\Delta U_i^*$, where it can be shown that

$$\Delta U_i^* = \frac{1}{\delta} \ln\left(1 + \delta\frac{\Delta y_i^*}{y_i^*}\right) - \sum_{l=1}^{F} \beta_l^i \ln\left(1 + \delta\frac{\Delta\omega_l^*}{\omega_l^*}\right)$$

where $\Delta\omega^* \equiv \Delta y^* AB$ and $\beta_l^i \equiv \sum_{j=1}^{G} \alpha_{ij}\beta_{jl}$ satisfies $\sum_{l=1}^{F} \beta_l^i = 1$ and $\sum_{l=1}^{F} y_i^* \beta_l^i = \omega_l^*$. Since for all $i \in \mathcal{H}$

$$U_i^* + \delta\Delta U_i^* > U_i$$

it follows that for δ sufficiently small we will still have $U_i^* + \delta\Delta U_i^* > U_i$ for all $i \in \mathcal{H}$. The perturbed income distribution $y^* + \delta\Delta y^*$ can then be supported with a uniform tax regime $\tau^* + \Delta\tau^*, \bar{y}^* + \delta\Delta\bar{y}^*$ using Theorem 5. Letting $\delta \to 0$

we have $\Delta U_i^* \to dU_i^*$ where

$$dU_i^* = \frac{\Delta y_i^*}{y_i^*} - \sum_{l=1}^{F} \beta_l^i \frac{\Delta \omega_l^*}{\omega_l^*}$$

which determines who gains and benefits as we begin to move away from y^* in the direction of Δy^*. Since $\sum_{l=1}^{F} y_i^* \beta_l^i = \omega_l^*$ it follows that $\sum_{l=1}^{F} y_i^* dU_i^* = 0$ and so there will be households with $dU_i^* > 0$ and some households with $dU_i^* < 0$.

5 A Numerical Example

Here we consider a numerical example to show how an optimal tax reform would actually work. Consider an economy with $H = 4$ households, $G = 4$ goods, and $F = 2$ factors of production. Obviously the model can be easily scaled up to allow for much larger values of H, G, and F.

The matrices A, B, E for this economy are assumed to be

$$A = \begin{bmatrix} 0.6 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.2 & 0.6 \end{bmatrix}, B = \begin{bmatrix} 0.2 & 0.8 \\ 0.5 & 0.5 \\ 0.7 & 0.3 \\ 0.8 & 0.2 \end{bmatrix}, E = \begin{bmatrix} 0.60 & 0.20 & 0.20 & 0.00 \\ 0.25 & 0.30 & 0.40 & 0.05 \end{bmatrix}.$$

In this example Household 1 is relatively rich, owning 60% of factor 1 and 25% of factor 2. Household 4 is relatively poor, owning none of Factor 1 and only 5% of Factor 2. Households 2 and 3 might be thought of as the middle class.

We first compute the competitive equilibrium with no taxation. We have

$$C \equiv ABE = \begin{bmatrix} 0.3795 & 0.2630 & 0.3260 & 0.0315 \\ 0.4390 & 0.2460 & 0.2920 & 0.0230 \\ 0.4775 & 0.2350 & 0.2700 & 0.0175 \\ 0.4915 & 0.2310 & 0.2620 & 0.0155 \end{bmatrix}$$

so that computing the unique left-hand eigenvector $y = yC$ with $y\iota_H = 1$ yields

$$\begin{aligned} y &= [y_i] = \begin{bmatrix} 0.4265 & 0.2496 & 0.2992 & 0.0248 \end{bmatrix} \\ v &= yA = \begin{bmatrix} 0.3132 & 0.2674 & 0.277 & 0.1423 \end{bmatrix} \\ \omega &= vB = \begin{bmatrix} 0.5041 & 0.4959 \end{bmatrix}. \end{aligned}$$

Rich Household 1 receives 43% of total income, the middle class receive roughly 55% of total income, while poor Household 4 only receives 2.5% of total income. This leads to the allocation

$$\begin{aligned} Q &= [q_{ij}] = \left[\frac{y_i \alpha_{ij}}{v_j}\right] = \begin{bmatrix} 0.8169 & 0.3189 & 0.1539 & 0.2997 \\ 0.0797 & 0.5599 & 0.1802 & 0.1754 \\ 0.0955 & 0.1119 & 0.6480 & 0.4205 \\ 0.0079 & 0.0093 & 0.0179 & 0.1045 \end{bmatrix} \\ K &= [K_{jl}] = \left[\frac{v_j \beta_{jl}}{\omega_l}\right] = \begin{bmatrix} 0.1243 & 0.5053 \\ 0.2653 & 0.2697 \\ 0.3847 & 0.1676 \\ 0.2258 & 0.0574 \end{bmatrix} \end{aligned}$$

with utility for each household given by

$$\begin{bmatrix} U_1 & U_2 & U_3 & U_4 \end{bmatrix} = \begin{bmatrix} -1.7958 & -2.4224 & -2.1909 & -4.6378 \end{bmatrix}. \quad (28)$$

5.1 Introducing Inefficient Taxation

Now we impose the following inefficient tax regime

$$\tau_d = diag \begin{bmatrix} 0.5 & 0.15 & 0.05 & 0.0 \end{bmatrix} \text{ and } \bar{y} = \begin{bmatrix} 0 & 0.2 & 0.2 & 0.6 \end{bmatrix}.$$

This tax regime taxes Good 1 the heaviest (the good that takes up the greatest budget share for rich Household 1) with a tax rate of 50%, while it leaves Good 4 untaxed (the good that takes up the greatest budget share of poor Household 4). Tax revenue is primarily given to poor Household 4, who gets 60% of tax revenue, while rich Household 1 gets no tax revenue.

We have

$$(I - A(I - \tau_d)BE)^{-1} = \begin{bmatrix} 2.364 & 0.7704 & 0.9162 & 0.0729 \\ 1.762 & 1.966 & 1.141 & 0.0873 \\ 1.894 & 1.010 & 2.185 & 0.0873 \\ 1.943 & 1.025 & 1.199 & 1.087 \end{bmatrix}$$

so that from (15) we have

$$\tau = \left(\bar{y}(I - A(I - \tau_d)BE)^{-1}\iota_4\right)^{-1} = 0.193$$

and tax revenue amounts to about 19% of all income. We then have

$$\begin{aligned} y &= \frac{\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}}{\bar{y}\left(I - A\left(I - \tau_d\right)BE\right)^{-1}\iota_4} = \begin{bmatrix} 0.3663 & 0.2337 & 0.2673 & 0.1327 \end{bmatrix} \\ v &= yA = \begin{bmatrix} 0.2832 & 0.2535 & 0.2703 & 0.1931 \end{bmatrix} \\ \omega &= v\left(I - \tau_d\right)B = \begin{bmatrix} 0.4704 & 0.3366 \end{bmatrix}. \end{aligned}$$

With this tax regime poor Household $4's$ share of income has risen from 2.5% to 13.3%. The inefficient tax allocation is then

$$Q = [q_{ij}] = \left[\frac{y_i \alpha_{ij}}{v_j}\right] = \begin{bmatrix} 0.7762 & 0.289 & 0.1355 & 0.1897 \\ 0.0825 & 0.5531 & 0.1729 & 0.1210 \\ 0.0944 & 0.1055 & 0.5934 & 0.2769 \\ 0.0469 & 0.0524 & 0.0982 & 0.4124 \end{bmatrix} \tag{29}$$

and

$$K = [K_{jl}] = \left[\frac{v_j\left(1 - \tau_j\right)\beta_{jl}}{\omega_l}\right] = \begin{bmatrix} 0.0602 & 0.3365 \\ 0.2291 & 0.3200 \\ 0.3822 & 0.2288 \\ 0.3285 & 0.1147 \end{bmatrix}. \tag{30}$$

Utility for the four households is

$$\begin{bmatrix} U_1 & U_2 & U_3 & U_4 \end{bmatrix} = \begin{bmatrix} -2.1316 & -2.4498 & -2.239 & -2.888 \end{bmatrix}. \tag{31}$$

Subtracting utility in (28) and (31) yields the change in utility ΔU_i for each household

$$\begin{bmatrix} \Delta U_1 & \Delta U_2 & \Delta U_3 & \Delta U_4 \end{bmatrix} = \begin{bmatrix} -0.34 & -0.027 & -0.048 & 1.75 \end{bmatrix}.$$

It can be shown that ΔU_i can be interpreted as the proportionate increase in overall consumption needed to make households indifferent between the two allocations. Using this metric rich Household 1 is the biggest loser, and experiences a welfare loss equivalent 34% of consumption. The two middle class households have a much more modest reduction in welfare of the order of 3% to 5% of consumption. Poor Household 4 is the big winner, experiencing a welfare gain equivalent to 175% of consumption.

5.2 Tax Reform

We now seek to find a Pareto dominating efficient allocation $\{Q^*, K^*\}$, as well as a tax regime that supports $\{Q^*, K^*\}$. To this end we calculate the required endowment matrix E^* from (30) and (29) as

$$E^* = K^T Q^T = \begin{bmatrix} 0.2271 & 0.2375 & 0.3476 & 0.1878 \\ 0.4064 & 0.2582 & 0.2331 & 0.1023 \end{bmatrix}$$

so that

$$C^* \equiv ABE^* = \begin{bmatrix} 0.3401 & 0.2506 & 0.2755 & 0.1339 \\ 0.3096 & 0.2470 & 0.2949 & 0.1485 \\ 0.2898 & 0.2448 & 0.3075 & 0.1579 \\ 0.2827 & 0.2439 & 0.3121 & 0.1613 \end{bmatrix}.$$

Computing the y^* that satisfies $y^* = y^* C^*$ and $y^* \iota_H = 1$ yields

$$\begin{aligned} y^* &= \begin{bmatrix} 0.3092 & 0.2470 & 0.2952 & 0.1487 \end{bmatrix} \\ v^* &= y^* A = \begin{bmatrix} 0.2546 & 0.2544 & 0.2872 & 0.2038 \end{bmatrix} \\ \omega^* &= v^* B = \begin{bmatrix} 0.5422 & 0.4578 \end{bmatrix}. \end{aligned}$$

This leads to the allocation $\{Q^*, K^*\}$ given by

$$\begin{aligned} Q^* &= [q^*_{ij}] = \left[\frac{y^*_i \alpha_{ij}}{v^*_j}\right] = \begin{bmatrix} 0.7287 & 0.2431 & 0.1077 & 0.1517 \\ 0.0970 & 0.5825 & 0.1720 & 0.1212 \\ 0.1160 & 0.1160 & 0.6168 & 0.2896 \\ 0.0584 & 0.0584 & 0.1035 & 0.4376 \end{bmatrix} \\ K^* &= [K^*_{jl}] = \left[\frac{v^*_j \beta_{jl}}{\omega^*_l}\right] = \begin{bmatrix} 0.0939 & 0.4449 \\ 0.2346 & 0.2779 \\ 0.3707 & 0.1882 \\ 0.3008 & 0.0891 \end{bmatrix} \end{aligned}$$

with utility

$$\begin{bmatrix} U^*_1 & U^*_2 & U^*_3 & U^*_4 \end{bmatrix} = \begin{bmatrix} -2.094 & -2.436 & -2.224 & -2.872 \end{bmatrix}. \qquad (32)$$

Subtracting utility U_i in (31) from utility U^*_i in (32) gives ΔU^*_i as

$$\begin{bmatrix} \Delta U^*_1 & \Delta U^*_2 & \Delta U^*_3 & \Delta U^*_4 \end{bmatrix} = \begin{bmatrix} 0.038 & 0.014 & 0.015 & 0.016 \end{bmatrix}.$$

All households are better off, as Theorem 4 predicts. Rich Household 1 gains the most, with an increase in welfare equivalent to a 3.8% in overall consumption. But the other three households gain too, with an increase in welfare equivalent to about 1.5% of overall consumption. If one wanted rich Household 1 to benefit less, and the other households to benefit more, one could perturb y^* as $y^* + \delta \Delta y^*$, as discussed in the Section 4.4.

We now seek a uniform tax regime with the minimal tax rate $\tau^*_{\min}$ that support this efficient allocation. We have

$$\hat{y} \equiv y^* ABE = \begin{bmatrix} 0.4345 & 0.2473 & 0.2946 & 0.0237 \end{bmatrix}.$$

Since $\hat{y} \neq y^*$, we have $\tau^*_{\min} > 0$, and so our tax reform will still require a role for the government. To calculate the minimum tax rate $\tau^*_{\min}$ we have

$$\begin{bmatrix} \frac{y_1^*}{\hat{y}_1} & \frac{y_2^*}{\hat{y}_2} & \frac{y_3^*}{\hat{y}_3} & \frac{y_4^*}{\hat{y}_4} \end{bmatrix} = \begin{bmatrix} 0.843 & 0.945 & 0.907 & 5.611 \end{bmatrix}$$

so that

$$\tau^*_{\min} \equiv 1 - \min_{i \in \mathcal{H}} \frac{y_i^*}{\hat{y}_i} = 1 - 0.843 = 0.157.$$

Thus

$$\bar{y}^* = \hat{y} + \frac{y^* - \hat{y}}{\tau^*_{\min}} = \begin{bmatrix} 0 & 0.16 & 0.12 & 0.72 \end{bmatrix}.$$

This efficient uniform tax regime has a tax rate of 16%, lower than the 19% overall tax rate of the inefficient tax regime. In the transition to the efficient tax regime, Rich Household 1 still receives no share of tax revenue. The middle class sees a modest reduction in their tax shares from 20% each to 16% and 12%. Poor Household 4 is helped by an increase in its tax share from 60% to 72%.

6 Conclusions

In this paper we have not only shown that uniform taxation is efficient, but we have shown how to implement it in a way that makes everybody in the economy strictly better off.

One might object that we have paid a too high cost for these results by assuming Cobb-Douglas functional forms; that more general functional forms that allow budget and factor shares to vary would provide more accurate predictions.

We have two responses to this objection. The first is that there is little or no evidence that *any CGE* model makes accurate numerical predictions. Instead of six digit accuracy, the goal of *CGE* analysis is to provide qualitative insight and suggest orders of magnitude. Given these largely qualitative goals, why insist on models with functional forms that make the model difficult to solve, and where the equilibrium may not be unique? The point is to give the traveller a map, and often that map need only be a quick sketch. Furthermore the associated computational difficulties with more general *CGE* models make it intractable to consider anything except the most basic levels of heterogeneity, and so this type of analysis is likely to miss out on the more fundamental issue: how to deal with heterogeneity. The map does need to be accurate, but it should point out the dangerous parts of the route.

The second response is that the model presented in this paper is in fact much more general than it might first appear. In Sampson (2013) it is shown that there is no possible data set or policy experiment that could refute the model used in this paper. This is because the model allows for an unlimited number of households H, goods G, and factors F. By appropriately labelling these objects by state of the world, time period, and location, and then matching budget and factor shares, it is possible to fit the model to any data set or to the results of any policy experiment. To turn the criticism of our model around, based on Occam's razor our model is *better* than the apparently more general *CGE* tax models that use more general functional forms.

7 References

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