Learning about New Eras

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Abstract

After World War II many feared the American economy would slip back into the Great Depression. This paper attempts to estimate the point in time when enough post-war data had been collected to convince people that in fact the post-war economy would be characterized by growth and stability. Using Bayesian methods a welfare measure of the cost of post-war uncertainty is constructed. Calculations show that post-war uncertainty was significant until the beginning of the 1970's. This suggests that it may be some time before the uncertainty regarding the current 2008 financial crisis is resolved.

1 Introduction

> *Indeed, almost nobody in America expected this. If anything, the lurking fear had been that the war years might end up being the high point of people's lives. There was little reason to think life would be much different after the war from the way it had been before. With all the veterans returning home looking for jobs, most people figured America would return to the Depression.*[1]

A non-ergodic regime change occurs when the economy enters a regime it has never been in before, from one it will never return to again. An important implication of this fact is that the historical record does not reveal the law-of-motion of the new regime. By way of contrast, ergodic regime changes occur in Hamilton's (1990) Markov switching model of the business cycle. Here the structure of the off-diagonal elements of the transition matrix (see the Appendix

[1] Garreau (1992, p. 358)

for discussion of ergodic and non-ergodic Markov chains) means that each state continually repeats itself: recessions are followed by expansions which are in turn followed by recessions. With ergodic regime changes the historical record reveals the underlying law-of-motion.

Most of the important economic regime changes in history are non-ergodic. Being unique and irreversible, they include epoch making inventions and discoveries such as electricity and the internet, the beginning and ending of major wars and financial crises such as the Great Depression, World War II, and the recent 2008 financial crisis. Since the new regime has never been seen before, the historical record does not reveal its law-of-motion. Instead agents must wait until they have collected enough data from the new regime to infer its law-of-motion. In the meantime the unknown law-of motion creates uncertainty.

From this it follows that a non-ergodic regime change sends a pulse of uncertainty through the economy that lasts as long as there is uncertainty about the new law-of-motion. This pulse is largest at the beginning of the new regime when agents have the least amount of data, and then declines as agents collect more and more data from the new regime. Given the historical importance and unexpectedness of many non-ergodic regime changes, the initial pulse of uncertainty is likely to be large. Given rational statistical learning, the decay is likely to be slow, at the parametric rate of $O\left(t^{-\frac{1}{2}}\right)$ (as with a standard error in econometrics) where t is the length of time the new regime has existed. In the meantime rational economic behavior will be reacting to this uncertainty, whether this is in asset pricing, investment, or saving (see for example Sampson 1998, 2003, 2013).

Of central importance is the length of time it takes for this pulse of uncertainty to decay to the point where it is negligible. This is important not only for understanding past economic events, but for understanding our current situation after the 2008 crisis. In 2008 the initial pulse of uncertainty was at an unprecedented level. Would the economy would move into a second Great Depression or would there be a quick recovery? What were the long term implications of the resulting government debt and monetary expansion? Today we have only five years of data from the post-2008 regime. At what point will we have collected enough data for this uncertainty to be resolved?

Rather than wait, we look at an earlier non-ergodic regime: the end of the Second World War and the beginning of the post-war era. Here it is reasonable to suppose that enough data was eventually collected to resolve the uncertainty about the nature of the post-war regime. But how long did it take?

By 1947 Americans had experienced three distinct regimes each with a particular law-of-motion: the prosperity of the 1920's, the Great Depression of the 1930's, and the war years from 1941-1945. Today we know that the post-war era would be characterized by relative growth and stability, but this was far from obvious in 1947. Would drastic reductions in military expenditure, credited by many with bringing the economy out of the Great Depression, now return the economy back to something like the Great Depression? What would be the impact of the return of ten million soldiers to the civilian workforce?

A number of papers appearing in the *Journal of Political Economy* reflected this uncertainty. Klein (1946) cites press reports in the autumn of 1945 that "Government economists predict 8 million unemployed by 1946" (the US labour force at the time was approximately 60 million). These forecasts turned out to be wrong, actual unemployment in 1946 was approximately 3 million, but Klein still thought the pessimistic forecasts might be correct by 1950. Woytinsky (1947) cites forecasts of unemployment as high as 20 million and calculates that, based on the econometric methodology used at the time, the forecast should be 6.5 million with a margin of error of 10 million. Woytinsky (1947) and the private sector forecasters referred to in Barnes (1948) were more optimistic. Barnes (1948) however shows that if the optimistic private sector forecasts had been correctly calculated, they in fact would have been in line with the pessimistic forecasts.

We adopt a methodology similar in spirit to Lucas's (1987) attempt to measure the potential welfare benefits of stabilization policy. A neoclassical agent is put in a position where in 1947 he is facing the future post-war era. We then allow him to year-by-year collect the actual US aggregate consumption stream. We then ask, using a welfare measure, at what point in time does this agent stop being concerned about the nature of the post-war era? The answer we obtain is that it took roughly 25 years, until about the year 1970, or roughly to the period when the rational expectations revolution took place in macroeconomics. If we use this as a guide for the post-2008 era, and make the optimistic assumption that the post-2008 era will resemble the post-war era, then it will still take something of the order of 20 years, or the year 2033, before the uncertainty surrounding 2008 is resolved.

2 The Methodology

The non-ergodic regime change occurs at $t = 0$, here the end of World War II which we take to begin in 1947. Let c_t be the log of real annual consumption. Following Lucas (1987) we assume that welfare at time t is

$$W_t = -E_t \left[\sum_{\tau=0}^{\infty} \exp\left(-\delta\tau + \theta c_{t+\tau}\right) \right] \tag{1}$$

where δ is the rate of discount and $1+\theta$ is the coefficient of relative risk aversion.

For the American economy a good approximation to the consumption process c_t for both the pre and post-war periods is a random walk with drift, and so we assume consumption growth $\Delta c_t \equiv c_t - c_{t-1}$ is *i.i.d.* and normally distributed as

$$\Delta c_t \sim N\left[\mu, \sigma^2\right]. \tag{2}$$

For the pre-war period the Q statistic for consumption growth with 5 lags is 2.79, the Jarque-Bera test statistic for normality is 1.06 and the LM test for an *ARCH(5)* is 4.86. Similarly for the 1947-1983 post-war period these statistics are $2.64, 2.38$ and 10.04 respectively. None of these is significant at the 5% level, indicating that the random walk assumption in (2) is reasonable in either the pre or post-war period.

To capture the uncertainty about the nature of the post-war economy we assume that the agent does not know μ and σ. In 1947 at $t = 0$ the true μ, σ given by μ_*, σ_* are drawn from a probability distribution with density $p(\mu, \sigma)$. Here μ_*, σ_* characterize the actual but unknown nature of the post-war economy. Today we know that $\mu_* \approx 0.02, \sigma_* \approx 0.02$; the American economy would be characterized by growth and stability. In 1947 people didn't know this. I assume instead that they know $p(\mu, \sigma)$, which then acts as a Bayesian prior.

At time t progresses people combine their prior beliefs $p(\mu, \sigma)$ with the post-war historical sample $S_t = \{c_0, c_1, \ldots, c_t\}$ to form a posterior distribution $p(\mu, \sigma | S_t)$ as

$$p(\mu, \sigma) \wedge S_t \Longrightarrow p(\mu, \sigma | S_t). \tag{3}$$

(The actual construction of the posterior $p(\mu, \sigma | I_t)$ is described in more detail in the next section.) By first conditioning on μ and σ it follows from (1) and (2) that welfare W_t can be expressed as

$$W_t(c_t) = -\exp(-\theta c_t) E_t\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right] \tag{4}$$

where $f(x) = 1/(1 - \exp(-x))$ and where the expectation E_t is over the unknown μ and σ using (3).

As t increases the increasing sample information $S_t = \{c_0, c_1, \ldots, c_t\}$ will result in a posterior $p(\mu, \sigma | S_t)$ that is more and more concentrated around the true nature of the post-war economy: μ_*, σ_*. There will thus be some date T by which time for all practical purposes we can assume that the nature of the post-war economy μ_*, σ_* is revealed as

$$E_T\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right] \approx f\left(\delta + \theta\mu_* - \frac{\theta^2}{2}\sigma_*^2\right).$$

Now to measure the economic importance of the uncertainty of the post-war regime at time t we calculate the permanent proportion of annual consumption y_t that the agent would be willing to give up in return for having the information at T; that is to have the nature of the post-war economy revealed. This y_t satisfies $V_t(c_t) = V_T(c_t - y_t)$ or

$$-\exp(-\theta c_t) E_t\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right] = -\exp(-\theta(c_t - y_t)) E_T\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right]$$

and so

$$y_t = \frac{1}{\theta}\ln\left(\frac{E_t\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right]}{E_T\left[f\left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2\right)\right]}\right). \tag{5}$$

A similar approach is used in Lucas (1987) to measure the potential welfare benefits of stabilization policy.

Here y_t measures the value in terms of annual consumption of knowing the true state of the American post-war economy. For example suppose $y_t = 0.2$ in 1950. Americans would then be willing to sacrifice twenty percent of annual consumption in return for knowing the true state of the post-war economy. Post-war uncertainty is clearly economically significant, and could be expected to have a huge impact on the asset markets, savings, and investment. Economists wanting to model this period would have to take this uncertainty into account. If instead $y_t = 0.001$ then Americans would only be willing to sacrifice one-tenth of one percent of annual consumption. Uncertainty about the post-war economy would then be economically insignificant, and could be safely ignored in economic models.

If one wishes one relate y_t to asset prices. Let π_t be the price-dividend ratio that comes from a standard asset pricing model based on (1), from which it can then be shown that

$$1 + \pi_t = E_t \left[f \left(\delta + \theta\mu - \frac{\theta^2}{2}\sigma^2 \right) \right]. \tag{6}$$

Using (6) we can express our welfare measure y_t as

$$y_t = \frac{1}{\theta} \ln \left(\frac{1 + \pi_t}{1 + \pi_T} \right) \approx \frac{1}{\theta} \frac{\pi_t - \pi_T}{\pi_T}.$$

Thus θy_t is the approximate percentage difference between the actual price dividend ratio π_t and the price dividend ratio π_T that would occur if the nature of the post-war regime were known. See Sampson (2003) for a more detailed explanation for how this can be used to explain stock market bubbles.

3 Results

For the consumption growth series Δc_t we use annual U.S. real per-capita consumption growth from 1891-1983 as shown in Figure 1.[2] From Figure 1 it is apparent that Δc_t behaves much differently after World War II than before.

The post-war period is defined as being 1947-1983 since calculating consumption growth for 1946 growth would require consumption in the war year 1945 and so we treat 1946 as a transition year. The pre-war sample from 1891-1939 is used as a reference for choosing plausible priors. To guide us in the choice of

[2] The real consumption series is the Kendrick series for 1890-1945 in 1929 prices and the NIPA series for 1945-1983 as found in the Appendix B constructed by N. Balke and R. Gordon in Gordon (1986). This is deflated by U.S. population which comes from the U.S. Department of Commerce, Bureau of the Census (1977) *Historical Statistics of the United States*, series A7 from 1919-1929, series A6 from 1930-1969, and from the *Economic Report of the President* (1989) from 1970-1983.

priors we calculate the ten-year rolling sample mean and standard deviation

$$\bar{\mu}_{t,10} = \frac{1}{10}\sum_{i=0}^{9}\Delta c_{t-i}, \ \bar{\sigma}_{t,10} = \sqrt{\frac{1}{10}\sum_{i=0}^{9}\Delta c_{t-i}^2 - \bar{\mu}_{t,10}^2} \tag{7}$$

shown in Figure 2.

Using Figure we chose three different priors designed to reflect the range of beliefs in 1946. The Very Pessimistic prior reflects a belief that the post-war economy would contract $\hat{\mu}_o = -0.01$ and be highly unstable $\hat{\sigma}_o = 0.1$, as given by the worst values of $\bar{\mu}_t^{10}$ and $\bar{\sigma}_t^{10}$ in 1932 during the Great Depression. The Pessimistic prior reflects a belief that the post-war economy would be stagnant $\hat{\mu}_o = 0$ and moderately unstable $\hat{\sigma}_o = 0.05$, as suggested by the values of $\bar{\mu}_{t,10}$ and $\bar{\sigma}_{t,10}$ at the end of the Great Depression in 1939. Finally the Optimistic 'prior with $\hat{\mu}_o = \hat{\sigma} = 0.02$ reflects a belief in stable growth as typified by the actual post-war experience.

We chose relatively diffuse priors with $t_0 = 10$ to insure that it is the post-war consumption data stream that reveals that the economy would both grow $\mu_* = 0.02$ and be stable $\sigma_* = 0.02$. If instead it was the government's announcement that it would commit to say Keynesian economics that revealed μ_*, σ_*, or some other deep structural understanding of the economy, then the prior might be $t_0 = 1000000$ with $\hat{\mu}_0 = \hat{\sigma}_0 = 0.02$. But then $y_t = 0$ and the calculation would not be interesting.

These as well as the other required parameters needed are given below.

Priors

	$\hat{\mu}_o$	$\hat{\sigma}_o$	t_0	θ	δ	$\mu_{\min}$	$\mu_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$
Very Pessimistic	−0.01	0.10	10	1	0.05	−0.03	0.05	0.005	0.15
Pessimistic	0.00	0.05	10	1	0.05	−0.01	0.10	0.005	0.10
Optimistic	0.02	0.02	10	1	0.05	−0.01	0.10	0.005	0.10

In this paper we take $T = 37$ or the year 1983 as the time by which the nature of the post-war has been revealed. Larger values of T are possible but in fact doing this would have little effect on the results.[3]

In Figures 3,4, and 5 we see how the posteriors for μ and σ are evolving as reflected by $\hat{\mu}_t$ and $\hat{\sigma}_t$. Both the Very Pessimistic and Pessimistic agents are learning that the post-war economy will be better than what they had first imagined.

In Figure 6 the welfare measure y_t in (5) is plotted for the Very Pessimistic, Pessimistic and Optimistic agents using $T = 37$ or 1983 as the comparison

[3] For $T = 37$ and the period 1947 to 1983 we have $\bar{\mu} = 0.021$, $\bar{\sigma} = 0.021$ while if instead it were $T^* = 57$ or 1947 to 2002 we would have $\bar{\mu} = 0.022$, $\bar{\sigma} = 0.018$.The correction to y_t that this would cause in Figure 6 would be 0.014 for each year.

year. All three agents are much less concerned about a return to the Great Depression as time progresses, but the learning process is slow. The agents are concerned about a return to the Great Depression well into the 1960's. For example in 1961 even the Optimistic agent has $y_t = 0.068$ so that he or she would be willing to sacrifice 6.8% of annual consumption to resolve the uncertainty concerning the post-war economy. The Very Pessimistic and Pessimistic agents are quite concerned with $y_t = 0.25$ (or 25% of annual consumption) for the Very Pessimistic agent and $y_t = 0.15$ (or 15% of annual consumption) for the Pessimistic agent.

The later part of the 1960's saw most of the worrying disappear as it became clear that the American economy would not slip back into the Great Depression. Thus by 1972 the Optimistic agent has stopped worrying with $y_t = 0.00$. The Pessimistic agent is still concerned with $y_t = 0.021$ (or 2.1% of annual consumption) but much less than in 1961. Even the Very Pessimistic agent has $y_t = 0.045$ (or 4.5% of annual consumption) which is less than y_t for the Optimistic agent in 1961.

4 Conclusion

Rational statistical learning is slow, at the parametric rate of $O\left(t^{-\frac{1}{2}}\right)$. When a non-ergodic regime change occurs it therefore takes some time before the new law-of-motion is understood. The results in this paper are conservative in the sense that we have assumed the functional form of the underlying model is known, and we have abstracted from the possibility of further non-ergodic regime changes.

It may be that in the real world agents learn faster than $O\left(t^{-\frac{1}{2}}\right)$, but this would imply they have better methods than those used by econometricians. Another possibility is that uncertainty is resolved faster than $O\left(t^{-\frac{1}{2}}\right)$ because of irrational learning. In this case agents might quickly converge on a false consensus model. This will likely create its own problems since as more-and-more data come in it will eventually become obvious even to irrational learners that their model is wrong.

In the case of the post-war economy our results suggest that it took until 1970 before uncertainty regarding the post-war regime was resolved. It may only be a coincidence, but it is surely interesting that the period around 1970 saw a regime change in macroeconomics from the old Keynesian regime to the more classical rational expectations and real business cycle regime. Perhaps this regime change took place because the younger macroeconomists at the time had enough data or experience to be confident in the nature of the post-war regime.

Today, after the 2008 crisis, we find ourselves in a situation similar to the early post-war era. The results in this paper suggest that it may be some time before this uncertainty is resolved, perhaps of the order of another 20 years.

5 Appendix

5.1 Priors and Posteriors

We now turn our attention to the construction of the posterior $p(\mu, \sigma | I_t)$ needed to calculate the welfare measure y_t. To insure that only economically meaningful realizations of the growth rate μ and volatility σ of the post-war economy are possible, we assume that the prior density $p(\mu, \sigma)$ is bounded as

$$\mu_{\min} \leq \mu \leq \mu_{\max}, \ \sigma_{\min} \leq \sigma \leq \sigma_{\max}.$$

Now we factor $p(\mu, \sigma)$ as $p(\mu, \sigma) = p(\mu|\sigma) \times p(\sigma)$ where $p(\mu|\sigma)$ and $p(\sigma)$ are defined as

$$\mu|\sigma \sim N\left[\hat{\mu}_o, \frac{\sigma^2}{t_0}, \mu_{\min}, \mu_{\max}\right] \tag{8}$$

and[4]

$$\frac{t_0 \hat{\sigma}_o^2}{\sigma^2} \sim \chi^2_{t_0}\left[\sigma_{\min}, \sigma_{\max}\right]. \tag{9}$$

The prior parameters $\hat{\mu}_o$ and $\hat{\sigma}_0$ determine the agent's belief of the most likely values of the growth rate μ and volatility σ of the post-war economy. The prior parameter t_0 determines the precision of the prior beliefs. Since $p(\mu, \sigma)$ is a conjugate prior it is equivalent to observing t_0 years of consumption growth. As t_0 increases the priors become more and more concentrated around $\hat{\mu}_0$ and $\hat{\sigma}_0$ with the limit $t_0 = \infty$ corresponding to $\sigma = \sigma_0$ and $\mu = \mu_0$ with certainty.

At time t people observe the historical post-war record of consumption $I_t = \{c_o, c_1, \ldots, c_t\}$. They then combine I_t with the prior information in (8) and (9) to form a posterior for μ and σ as

$$p(\mu, \sigma | I_t) = p(\mu|\sigma, I_t) \times p(\sigma | I_t)$$

where from standard Bayesian results $p(\mu|\sigma, I_t)$ is

$$\mu|\sigma \sim N\left[\hat{\mu}_t, \frac{\sigma^2}{t + t_o}, \mu_{\min}, \mu_{\max}\right] \tag{10}$$

and $p(\sigma | I_t)$ is

$$(t + t_0)\frac{\hat{\sigma}_t^2}{\sigma^2} \sim \chi^2_{t+t_0}\left[\sigma_{\min}, \sigma_{\max}\right]. \tag{11}$$

[4] Here $Z \sim N\left[\mu, \sigma^2, z_{\min}, z_{\max}\right]$ denotes a truncated normal with density

$$p_z(z) = c_1 \times \exp\left(-\frac{1}{2}z^2\right) \text{ for } z_{\min} \leq z \leq z_{\max}$$

and $\chi^2_{t_0}\left[w_{\min}, w_{\max}\right]$ denotes a truncated chi-squared with density

$$p_w(w) = c_2 \times w^{r/2-1} \exp\left(-\frac{1}{2}w\right) \text{for } w_{\min} \leq w \leq w_{\max}.$$

Here $\hat{\mu}_t$ and $\hat{\sigma}_t$, people's expectations of the most likely values of μ and σ, are

$$\hat{\mu}_t = \frac{t_0 \hat{\mu}_o}{t + t_o} + \frac{t \bar{\mu}_t}{t + t_o},\ \hat{\sigma}_t^2 = \frac{t_0 \hat{\sigma}_o^2 + t \bar{\sigma}_t^2}{t + t_0} + \frac{t_0 t \left(\hat{\mu}_o - \bar{\mu}_t\right)^2}{\left(t + t_0\right)^2} \tag{12}$$

where

$$\bar{\mu}_t = \frac{1}{t} \sum_{\tau=1}^{t} \Delta c_\tau,\ \bar{\sigma}_t^2 = \frac{1}{t} \sum_{\tau=1}^{t} \Delta c_\tau^2 - \bar{\mu}_t^2$$

are the sample mean and variance from the post-war record.

Now (10) and (11) determines E_t used to calculate the welfare measure y_t in (5) as

$$E_t \left[f \left(\delta + \theta\mu - \frac{\theta^2}{2} \sigma^2 \right) \right] = \int_{\mu_{\min}}^{\mu_{\max}} \int_{\sigma_{\min}}^{\sigma_{\max}} f \left(\delta + \theta\mu - \frac{\theta^2}{2} \sigma^2 \right) p \left(\mu, \sigma | S_t \right) d\mu d\sigma.$$

5.2 Monte Carlo Integration

For the i^{th} replication and $t + v > 0$, generate $t + v$ squared independent standard normals Z_{ij}, $j = 1, 2, \ldots, t + v$ and let $\chi^2_{i,t+v} = \sum_{j=1}^{t+v} Z_{ij}^2$, which has a chi-squared distribution with $t + v$ degrees of freedom. If $\sigma_{\min} \leq \sigma_i \leq \sigma_{\max}$ is satisfied where:

$$\sigma_i = \sqrt{\frac{(t + v)\, \hat{\sigma}_t^2}{\chi^2_{i,t+v}}}$$

then this is the i^{th} draw from the posterior distribution of σ given in (11). If the bounds $\sigma_{\min} \leq \sigma_i \leq \sigma_{\max}$ are not satisfied, this draw is rejected and a new draw from the χ^2 distribution is generated until the bounds are satisfied.

A draw from the conditional posterior of μ in (10) is generated by drawing a standard normal, say $\tilde{Z}_i$ and, if $\mu_{\min} \leq \mu_i \leq \mu_{\max}$ is satisfied, the i^{th} draw of μ will be:

$$\mu_i = \hat{\mu}_t + \tilde{Z}_i \frac{\sigma_i}{\sqrt{t + t_o}}.$$

If $\mu_i < \mu_{\min}$ or $\mu_i > \mu_{\max}$ this draw of $\tilde{Z}_i$ is rejected, and new draws are generated until $\mu_{\min} \leq \mu_i \leq \mu_{\max}$ is satisfied.

Once n draws of μ_i and σ_i for $i = 1, 2, ..., n$ have been generated then $E_t \left[f \left(\delta + \theta\mu - \frac{\theta^2}{2} \sigma^2 \right) \right]$ is consistently estimated by

$$\hat{E}_t \left[f \left(\delta + \theta\mu - \frac{\theta^2}{2} \sigma^2 \right) \right] \equiv \frac{1}{n} \sum_{i=1}^{n} f \left(\delta + \theta\mu_i - \frac{\theta^2}{2} \sigma_i^2 \right).$$

In the calculations above we used $n = 50,000$.

6 Markov Chains and Ergodicity

Consider an economy with two regimes: an expansion regime 1 and a recession regime 2. The transitions between the two regimes are governed by an ergodic Markov chain with transition matrix

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0.09 & 0.91 \end{bmatrix}.$$

What makes this transition matrix ergodic is that both off-diagonal elements are non-zero, thus insuring that both regimes occur infinitely often. This Markov chain has an equilibrium probability distribution of $p_1 = 0.9$, $p_2 = 0.1$ so that the economy is in an expansion 90% of the time and in a recession 10% of the time. An infinitely long historical record will have an infinite number of observations from both regimes and so reveal the dynamics of both regimes, as well as the above transition matrix.

A Markov chain with a non-ergodic regime is

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} 0.99 & 0.01 \\ 0 & 1 \end{bmatrix}.$$

If Regime 2 is that a war or invention occurs, and if the economy begins in Regime 1, then it will take on average 100 years for the war or invention to take place, after which there is no chance of returning to the pre-invention or pre-war regime. Here the economy remains in Regime 2 forever once the transition takes place. It is possible to instead have a never ending sequence of non-ergodic regime changes if the transition matrix takes the form

$$\begin{bmatrix} p_{11} & p_{12} & 0 & 0 & \cdots \\ 0 & p_{22} & p_{23} & 0 & \cdots \\ 0 & 0 & p_{33} & p_{34} & \cdots \\ 0 & 0 & 0 & p_{44} & \ddots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

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